

where $\mu, \mu_c \neq 1, \mu_c \neq (1 - \mu) / \gamma$.

Hence, $w_{lv} > 1$ when:

$$\mu_c > \frac{1 - \mu}{2(1 - \mu) - \gamma} = \mu_c^*$$

where $\gamma \neq 2(1 - \mu)$. Wages cannot be lower than 1 because in this case the traditional good would be produced in region v and not in region r .

On the contrary, when $0 < \mu_c \leq \mu_c^*$, the wages of unskilled workers in the core v must be equal to 1 if the traditional good is produced in the periphery r .

Therefore we may have the two following cases.

When $0 < \mu_c \leq \mu_c^*$, agglomeration of manufacturing firms in region v is an equilibrium if the ratio Q_{mir}/Q_{mir}^* from (41) is smaller than 1:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{(\tau^{2(\sigma-1)} - 1)(1-\mu_c\gamma-\mu)}{2} + 1 \right) < 1 \quad (51)$$

Otherwise, when $1 > \mu_c > \mu_c^*$, agglomeration of the manufacturing sector in region v is an equilibrium when:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{(1-\mu)(1-\mu_c)}{(1-\mu-\gamma)\mu_c} \right)^{-\sigma(1-\gamma-\mu)} \left[\left(\tau^{2(\sigma-1)} - 1 \right) (1 - \mu)(1 - \mu_c) + 1 \right] < 1 \quad (52)$$

Appendix B.

To prove that profits in a neighborhood of a long run equilibrium can be written as a function of the number of firms n

$$\pi_i = u(n) \quad (53)$$

it is necessary to determine the short run equilibrium, which is defined as a set of solutions to equations (54)-(58) below, once n_n and n_s are given. We express them in matrix form. To this end, variables without suffix r define vectors, variables with superscript \sim are 2x2 diagonal matrix with

the i -th element of the corresponding vector in position (i,i) and zeros off the diagonal. Matrix T

$$\text{is: } T = \begin{bmatrix} 1 & \tau^{1-\sigma} \\ \tau^{1-\sigma} & 1 \end{bmatrix}.$$

Substituting manufacturing prices p_r from (13), expenditures on the manufacturing good E_{mr} from (4), (5) and (10), manufacturing quantities Q_{mir} from (15), and production cost TC_{mir} from (9), into (12), (3), (20), (18), (19) and (21), for $r = n, s$, and given the normalizations of α and β , we obtain a system of 10 equations. The solutions of the system (54)-(58) is given by the set of the 10 “fast” variables $(\pi_{in}, \pi_{is}, p_{mn}, p_{ms}, w_{hn}, w_{hs}, w_{ls}, w_{ln}, H_n, H_s)$ for given values of the slow variables n_n and n_s .

In matrix form, the equilibrium is obtained by solving the following system:

- two manufacturing good market short run equilibrium conditions:

$$a + \sigma \tilde{a} \tilde{p}_m^{-\mu} \tilde{w}_l^{-1+\mu+\gamma} \tilde{w}_h^{-\gamma} \pi_i = \quad (54)$$

$$\begin{aligned} &= \mu_c \tilde{a}^\sigma \tilde{p}_m^{-\sigma\mu} \tilde{w}_l^{-\sigma(1-\mu-\gamma)} \tilde{w}_h^{-\sigma\gamma} T \tilde{p}_m^{\sigma-1} \left(\tilde{L} w_l + \tilde{H} w_h + \tilde{n} \pi_i \right) + \\ &+ \mu \tilde{a}^\sigma \tilde{p}_m^{-\sigma\mu} \tilde{w}_l^{-\sigma(1-\mu-\gamma)} \tilde{w}_h^{-\sigma\gamma} T \tilde{p}_m^{\sigma-1} \tilde{n} \left[(\sigma-1) \pi_i + \tilde{w}_l^{(1-\mu-\gamma)} \tilde{p}_m^\mu w_h^\gamma \right] \end{aligned}$$

- two composite good price indices:

$$0 = p_m^{1-\sigma} - T \tilde{a}^{(-1+\sigma)} \tilde{p}_m^{\mu(1-\sigma)} \tilde{w}_h^{\gamma(1-\sigma)} \tilde{w}_l^{(1-\gamma-\mu)(1-\sigma)} n \quad (55)$$

- two functions that express total wages of skilled workers in the two regions:

$$0 = \tilde{w}_h H - \gamma \tilde{n} \left[(\sigma-1) \pi_i + \tilde{p}_m^\mu \tilde{w}_l^{1-\gamma-\mu} w_h^\gamma \right] \quad (56)$$

- skilled labor market equilibrium condition together with the condition of equal real wages for the two regions:

$$\begin{bmatrix} \frac{w_{hn}}{p_{mn}^\mu} \\ H_n + H_s \end{bmatrix} = \begin{bmatrix} \frac{w_{hs}}{p_{ms}^\mu} \\ \bar{H} \end{bmatrix} \quad (57)$$

- two unskilled labor market conditions:

$$\begin{aligned} \tilde{w}_l \bar{L} &= (1 - \gamma - \mu) \tilde{n} \left[(\sigma - 1) \pi_i + \tilde{p}_m^\mu \tilde{w}_h^\gamma w_l^{1-\gamma-\mu} \right] + \\ &+ (1 - \mu_c) \Lambda (\tilde{w}_h H + \tilde{w}_l \bar{L} + \tilde{n} \pi_i) \end{aligned} \quad (58)$$

$$\text{where } \Lambda = \begin{bmatrix} \lambda_r & 1 - \lambda_v \\ 1 - \lambda_r & \lambda_v \end{bmatrix}$$

Let:

1. $x = (p_{mn}, p_{ms}, w_{hn}, w_{hs}, w_{ls}, w_{ln}, H_n, H_s)'$, a column vector of eight fast variables;
2. $y = (\pi'_i, x')'$ the column vector of the ten fast variables;
3. G_k be a function from R^{12} to R with continuous derivative in the neighborhood of a long run equilibrium (LRE), such that $G_k(y, n) = 0$, with $k = 1, 2, \dots, 10$ are the ten equations (54)-(58);
4. $G(y, n) \equiv (G_k(y, n))$.

If the $\det = \left[\frac{\partial G_k}{\partial y_l} \right]_* \neq 0$, where y_l is a generic element of y and $*$ means that the derivatives are evaluated at a LRE, equation $G(y, n) = 0$ allows us to define in a neighborhood of such LRE function u from R^2 to R^2 with continuous derivative such that

$$\pi_i = u(n)$$

The Jacobian matrix of u in a LRE is denoted by $\frac{\partial u}{\partial n}(n^*)$.

Finally, it should be noted that at the long run equilibrium values, that is, at long run equilibrium values of all fast and slow (n_n and n_s) variables, profits should be equal to zero ($\pi_{in} = \pi_{is} = 0$).

Appendix C.

In this appendix we show how we compute the Jacobian matrix evaluated at the long run equilibrium

$$J_1^* = \frac{\partial z}{\partial n}(n^*) = \delta M$$

where $M = \frac{\partial u}{\partial n}(n^*)$.

Define:

- the column vector $x = (p_{mn}, p_{ms}, w_{hn}, w_{hs}, w_{ls}, w_{ln}, H_n, H_s)'$;
- and the two functions f and g , where:

f is defined from R^{12} to R^2 and is derived from the two equations (54) in appendix B, and is such that

$$f(\pi_i, n, x) = 0$$

g is defined from R^{12} to R^8 and is derived from the eight equations (55)-(58), and is such that

$$g(\pi_i, n, x) = 0$$

Total differentials of f and g are respectively given by (59) and (60):

$$A d\pi_i + B dn + C dx = 0 \tag{59}$$

$$D d\pi_i + E dn + F dx = 0 \tag{60}$$

where matrices A, B, C, D, E and F are evaluated at symmetric equilibrium values, that can be computed from the system of equation (54)-(58) and are given below.

Computing dx from (60),

$$dx = -F^{-1}(D d\pi_i + E dn)$$

and substituting it into (59), yields:

$$M = \frac{\partial \pi_i}{\partial n} = -(-CF^{-1}D + A)^{-1}(-CF^{-1}E + B)$$

Long run symmetric equilibrium values, which can be obtained only if technological development levels are equal in the two regions, $a_n = a_s = 1$, are:

$$\begin{aligned} w_{lr} &= 1; & \pi_{ir} &= 0; & H_r &= \frac{\bar{H}}{2}; & w_{hr} &= \frac{2\gamma\mu_c\bar{L}}{H(1-\mu-\gamma\mu_c)}; \\ n_r &= (1 + \tau^{1-\sigma})^{-\frac{\mu}{1-\sigma+\mu\sigma}} \left(\frac{\mu_c\bar{L}}{1-\mu_c\gamma-\mu} \right)^{\frac{(1-\sigma)(1-\mu-\gamma)}{1-\sigma+\mu\sigma}} \left(\frac{2\gamma}{H} \right)^{\frac{\gamma(\sigma-1)}{1-\sigma+\mu\sigma}}, \\ p_{mr} &= (1 + \tau^{1-\sigma})^{\frac{1}{1-\sigma+\mu\sigma}} \left(\frac{\mu_c\bar{L}}{1-\mu_c\gamma-\mu} \right)^{\frac{1-\gamma\sigma}{1-\sigma+\mu\sigma}} \left(\frac{2\gamma}{H} \right)^{-\frac{\gamma\sigma}{1-\sigma+\mu\sigma}} \end{aligned}$$

where $r = n, s$.

Solutions are positive for $\mu_c < \frac{1-\mu}{\gamma}$.